

HKUST

MATH1013 Calculus IB

Final Examination (White Version)

Name: _____

7th Dec 2017

Student ID: _____

16:30-19:30

Lecture Section: _____

Directions:

- This is a closed book examination. **No Calculator is allowed in this examination.**
- DO NOT open the exam until instructed to do so.
- Turn off all phones and pagers, and remove headphones. All electronic devices should be kept in a bag away from your body.
- Write your name, ID number, and Lecture Section in the space provided above, and also in the **Multiple Choice Item Answer Sheet** provided.
- **Make sure you have written and marked your ID number correctly in the I.D. No. Box in the Multiple Choice Item Answer Sheet.**
- DO NOT use any of your own scratch paper, and do not take any scratch paper away from the examination venue.
- When instructed to open the exam, please check that you have **10** pages of questions in addition to the cover page. Two blank pages attached at the end can be torn off and used as scratch paper.
- Answer all questions. Show an appropriate amount of work for each short or long problem. If you do not show enough work, you will get only partial credit.
- You may write on the backside of the pages, but if you use the backside, clearly indicate that you have done so.
- **Cheating is a serious violation of the HKUST Academic Code. Students caught cheating will receive a zero score for the examination, and will also be subjected to further penalties imposed by the University.**

Please read the following statement and sign your signature.

I have neither given nor received any unauthorized aid during this examination. The answers submitted are my own work.

I understand that sanctions will be imposed, if I am found to have violated the University's regulations governing academic integrity.

Student's Signature :

Question No.	Points	Out of
Q. 1-18		54
Q. 19		10
Q. 20		10
Q. 21		15
Q. 22		11
Total Points		100

Part I: Answer all of the following multiple choice questions.

- Mark your MC answers to the boxes in the **Multiple Choice Item Answer Sheet** provided.
- Mark only one answer for each MC question. Multiple answers entered for each single MC question will result in a 3 point deduction.

Write also your MC question answers in the following boxes for back up use and for paper checking session only. The grading will be based on the answers you mark on the MC item answer sheet.

Question	1	2	3	4	5	6	7	8	9	10
Answer										
Question	11	12	13	14	15	16	17	18		
Answer										

Each of the MC questions is worth 3 points.

1. What is the color version of your exam? No point will be given to you for this question if you do not choose the color version of your exam correctly, or do not mark your ID number correctly in the MC answer form. (Check your ID marking in the MC form right now!!)

(a) Green (b) Orange (c) White (d) Yellow (e) None of the previous

2. Find the limit $\lim_{x \rightarrow -1^+} \frac{(x+1)(x+3)}{|x^2-1|}$.

(a) -2 (b) -1 (c) 1 (d) 2 (e) does not exist

3. Find the constant k that makes the following function continuous everywhere.

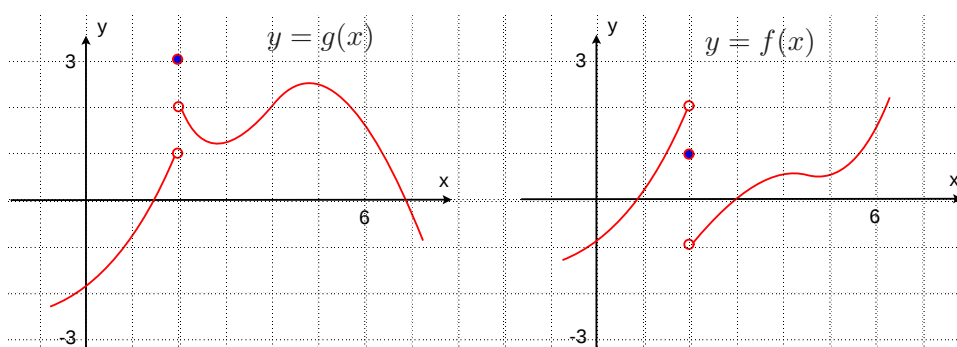
$$f(x) = \begin{cases} e^x + \frac{\sin 2x}{x} & \text{if } x < 0 \\ 3 \sin x + ke^x & \text{if } x \geq 0 \end{cases}$$

(a) does not exist (b) 1 (c) 2 (d) 3 (e) 4

4. Which of the following is a horizontal asymptote of the function $f(x) = \frac{2e^{2x} + 3x}{e^{3x} + 2x}$?

- (a) $y = \frac{2}{3}$ (b) $y = \frac{3}{2}$ (c) $y = 1$ (d) $y = 2$ (e) None of the previous

5. Given the graphs of $y = f(x)$ and $y = g(x)$ as shown below, find $\lim_{x \rightarrow 2^+} f(g(x))$.



- (a) 0 (b) 1 (c) 2 (d) -1 (e) does not exist

6. Let $f(x) = 3^{x^2}$. Find the derivative $f'(1)$.

- (a) 1 (b) 6 (c) $2 \ln 3$ (d) $3 \ln 3$ (e) $6 \ln 3$

7. Find the slope of the tangent line to the curve defined by the equation $x^2 - xy + y^2 = 4$ at the point $(0, 2)$.

- (a) $\frac{1}{2}$ (b) $\frac{3}{2}$ (c) 1 (d) -1 (e) $-\frac{1}{2}$

8. Find the limit $\lim_{x \rightarrow 0^+} x^{\frac{\ln 2}{1 + \ln x}}$.

- (a) $\ln 2$ (b) $\ln 3$ (c) 0 (d) 1 (e) 2

9. How many real roots does the equation $x^5 - 5x^2 + 2 = 0$ have?

- (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

10. How many inflection points does the function f have if its derivative function is

$$f'(x) = (x+1)^2(x-3)^3 ?$$

- (a) 1 (b) 2 (c) 3 (d) 4 (e) 5

11. Find the absolute minimum of the function $f(x) = \sin x + \cos x$ on the interval $0 \leq x \leq \frac{\pi}{2}$.

- (a) 0 (b) 1 (c) $\frac{1}{\sqrt{2}}$ (d) $\sqrt{2}$ (e) $\frac{\sqrt{3}}{2}$

12. Which of the following gives the antiderivatives of $\frac{1-x^2}{(1+x^2)^2}$?

- (a) $\frac{x}{1+x^2} + C$ (b) $\frac{1}{1+x^2} + C$ (c) $\frac{1-x}{1+x^2} + C$
(d) $\frac{1}{(1+x^2)^2} + C$ (e) $\frac{x}{(1+x^2)^2} + C$

13. A car is moving at 20 m/s, and starts to brake at time $t = 0$. Suppose that the velocity of the car is given by the function

$$v(t) = 20 - 5t^2 \quad (\text{m/s})$$

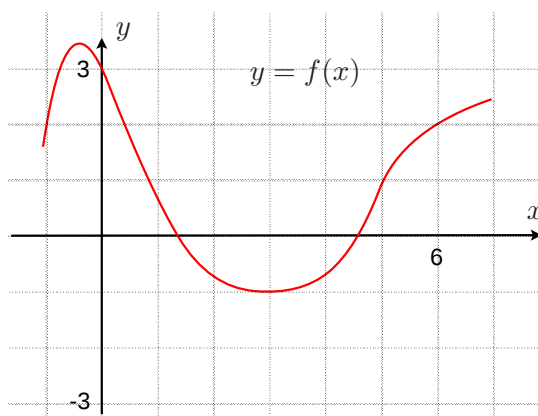
until it comes to a stop. Determine the distance the car has traveled since braking time when it comes to a stop.

- (a) $\frac{20}{3}$ m (b) $\frac{40}{3}$ m (c) $\frac{50}{3}$ m (d) $\frac{80}{3}$ m (e) $\frac{110}{3}$ m

14. Find the value of the definite integral $\int_0^{\ln 6} 3e^x \sqrt{e^x + 3} dx$.

- (a) 12 (b) 18 (c) 24 (d) 32 (e) 38

15. Given the graph of the function $y = f(x)$ as shown below, find $\int_0^6 |f'(x)| dx$.



- (a) -1 (b) 0 (c) 3 (d) 5 (e) 7

16. Find the limit $\lim_{n \rightarrow \infty} \left[\frac{1}{\sqrt{n^2 + 3n}} + \frac{1}{\sqrt{n^2 + 6n}} + \cdots + \frac{1}{\sqrt{n^2 + 3n^2}} \right] = \lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{\sqrt{n^2 + 3kn}}.$
- (a) $\frac{2}{3}$ (b) $\frac{3}{2}$ (c) $\frac{1}{2}$ (d) 1 (e) 2

17. If a continuous function f satisfies the equation

$$\int_0^x f(t)dt + \int_0^{x^2} tf(t)dt = 3x^3 ,$$

find the function value $f(1)$.

- (a) 0 (b) 1 (c) 2 (d) 3 (e) 4

18. The function f defined by

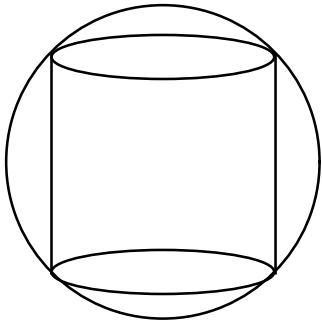
$$f(x) = \int_{\pi}^x \frac{2 + e^{\sin t}}{3} dt$$

has an inverse function denoted by f^{-1} . Find the derivative $(f^{-1})'(0)$.

- (a) 1 (b) 2 (c) $\frac{1}{3}$ (d) $\frac{2}{3}$ (e) $\frac{1}{2}$

Part II: Answer each of the following questions.

19. [10 pts] Find the dimensions (i.e., base radius and height) of the right circular cylinder of largest surface area that can be inscribed in a sphere of radius 4 m. *Justify your answer for full credit.*



20. [10 pts] A particle is moving along the curve defined by the equation

$$yx^3 + y^2 = 2x^2$$

Suppose that the x -coordinate is increasing at the rate of 6 units/s when the particle is at the point $(1, 1)$.

- (a) At what rate is the y -coordinate of the point changing at that instant? [8 pts]

- (b) Is the particle rising or falling at that instant? [2 pts]

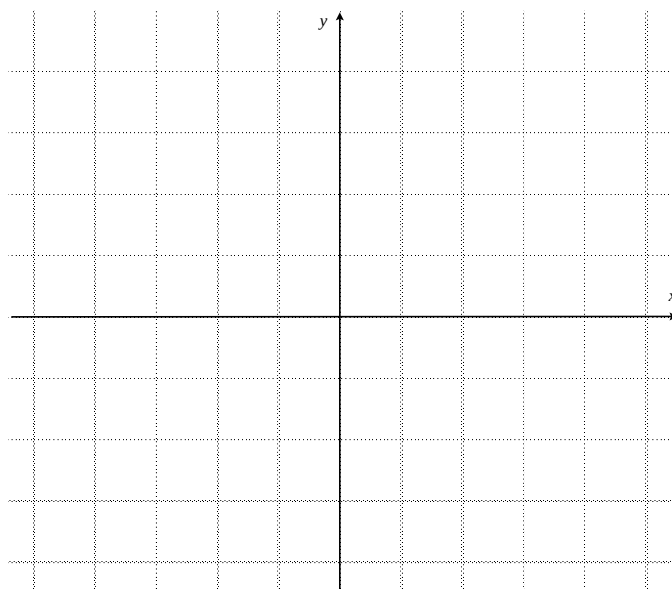
21. [15 pts] Consider the function $f(x) = 2 + \frac{2}{x^2 - 4}$ on the interval $-\infty < x < \infty$, with

$$f'(x) = \frac{-4x}{(x^2 - 4)^2} \quad \text{and} \quad f''(x) = \frac{12x^2 + 16}{(x^2 - 4)^3}.$$

- (a) Find the intervals on which f is increasing and the intervals on which f is decreasing. Show your work for full credit. [4 pts]

- (b) Find the intervals on which f is concave up. Justify your answer for full credit. [3 pts]

- (c) Using appropriate scales on the axes in the given grid, sketch the graph of f , and indicate all its local maximum or minimum point(s), inflection point(s) and asymptote(s). [8 pts]



22. [11 pts] Let f be a continuous function on the interval $0 \leq x \leq a$.

(a) Show that $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ by the substitution rule. [2 pts]

(b) Explain why $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ by using the Riemann sum definition (limit definition) of definite integral. [4 pts]

(c) Use the result of (a) or otherwise, evaluate $\int_0^{\frac{\pi}{2}} \frac{3 \cos x - \sin x}{\cos x + \sin x} dx$. [5 pts]